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Data assimilation method for the ocean circulation model NEMO and its application for the calculation of ocean characteristics in the Arctic Zone of Russia

K. Belyaev*, A. Kuleshov**, I. Smirnov***

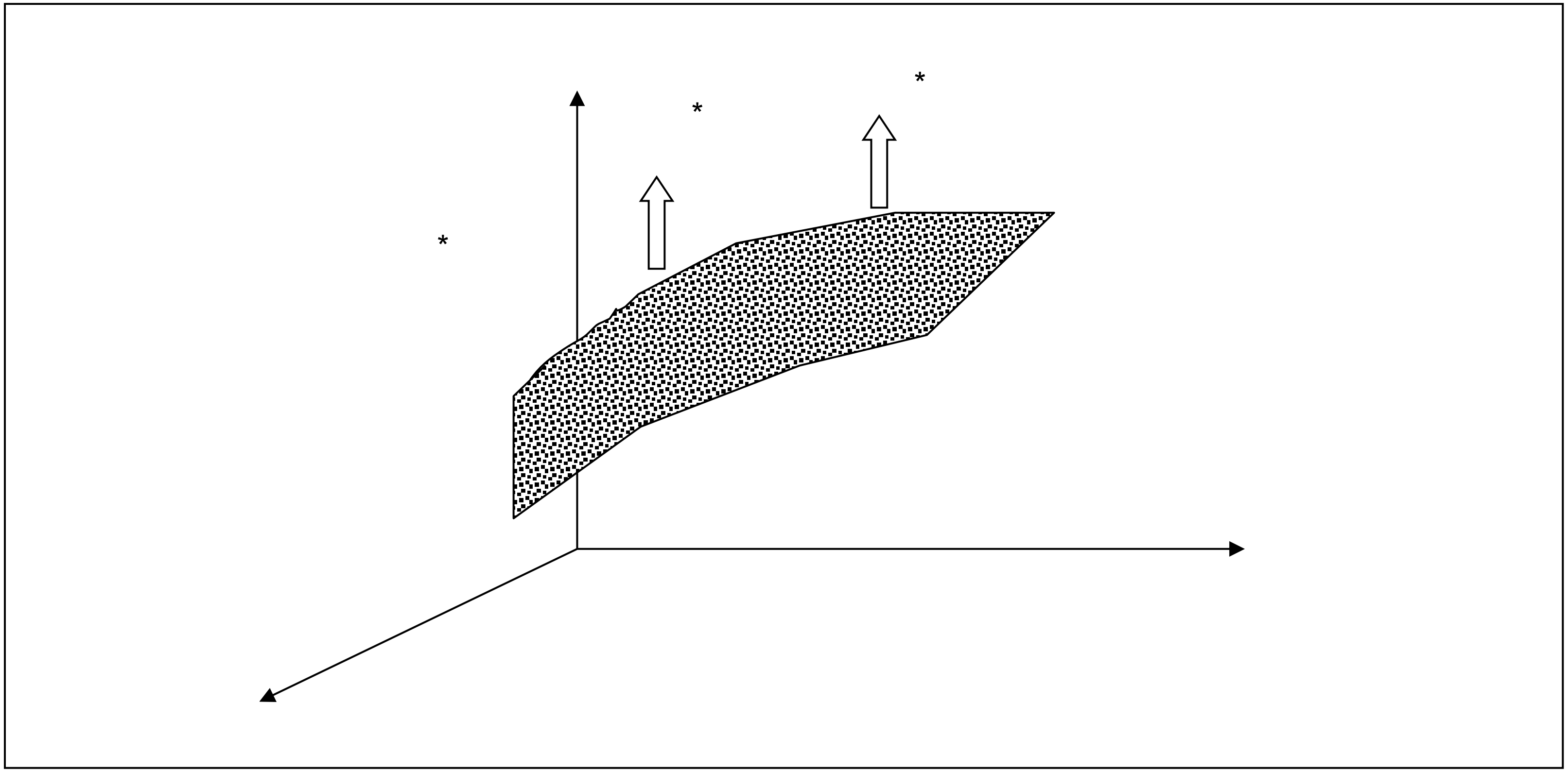
* Shirshov Institute of Oceanology ofRAS
** Keldysh Institute of Applied Mathematics of RAS
*** Lomonosov Moscow State University

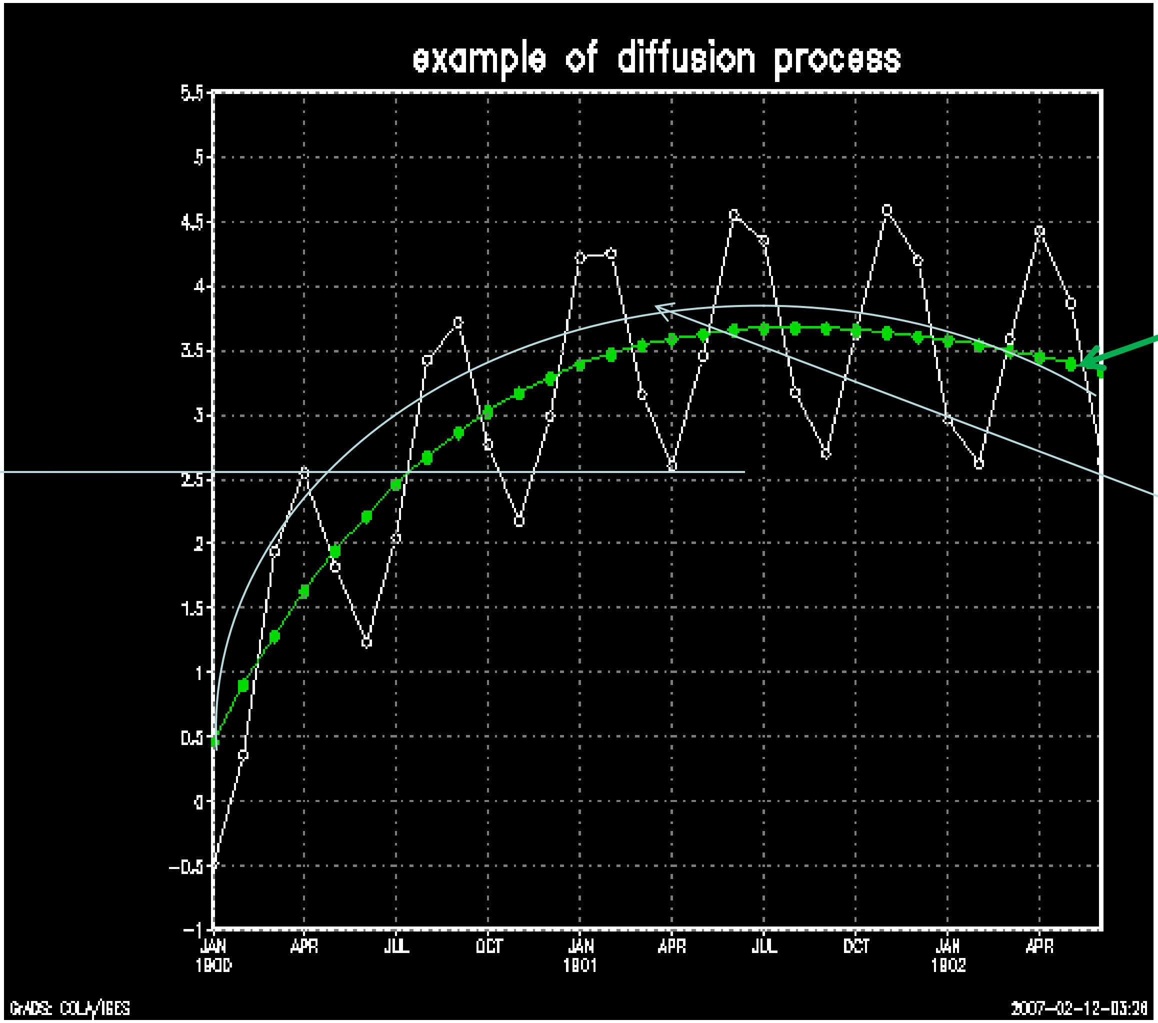
Data assimilation problem
when the model state vector is corrected
by observations is considered.

The goal of investigation:
Application of a new data assimilation method using satellites observations for the ocean circulation model NEMO.

Geometric interpretation

The main idea of all data assimilation methods is the best approximation of the model fields to observed data maintaining the model conserved lows.





$$a(t) = (I - KH)\Lambda$$

$$a(t) = \Lambda(t)$$

Ocean circulation model NEMO

NEMO – Nucleus for European Modelling of the Ocean

- created in Pierre Simon Laplace Institute (Paris)
- consists of parallel executed several modules

NEMO structure

• NEMO-OCE solves the basic Navier-Stokes equations and models ocean thermodynamics.

•NEMO-ICE simulates the dynamics of sea ice, including changes in ice thickness and salinity.

•NEMO-TOP models transport routes and biogeochemical processes

• Interface block XIOS

$\mathbf{U} = \mathbf{U_h} + w\mathbf{k}$

Ocean Model NEMO-OCE

Navier-Stokes equations with Boussinesq assumptions made

$$\frac{\partial \mathbf{U_h}}{\partial t} = \left[(\nabla \times \mathbf{U}) \times \mathbf{U} + \frac{1}{2} \nabla (\mathbf{U}^2) \right]_{\mathbf{h}} - f\,\mathbf{k} \times \mathbf{U_h} - \frac{1}{\rho_0} \nabla_{\mathbf{h}} p + \mathbf{D^U} + \mathbf{F^U}$$

$$\frac{\partial p}{\partial z} = -\rho g, \qquad \nabla \cdot \mathbf{U} = 0, \qquad \rho = \rho(T, S, p)$$

$\mathbf{U_h}$ is the horizontal velocity $\mathbf{U} = \mathbf{U_h} + w\mathbf{k}$, w is the vertical component of velocity

ρ is water density, ρ_0 is horizontal average density,

g is gravity parameter f is Coriolis acceleration,

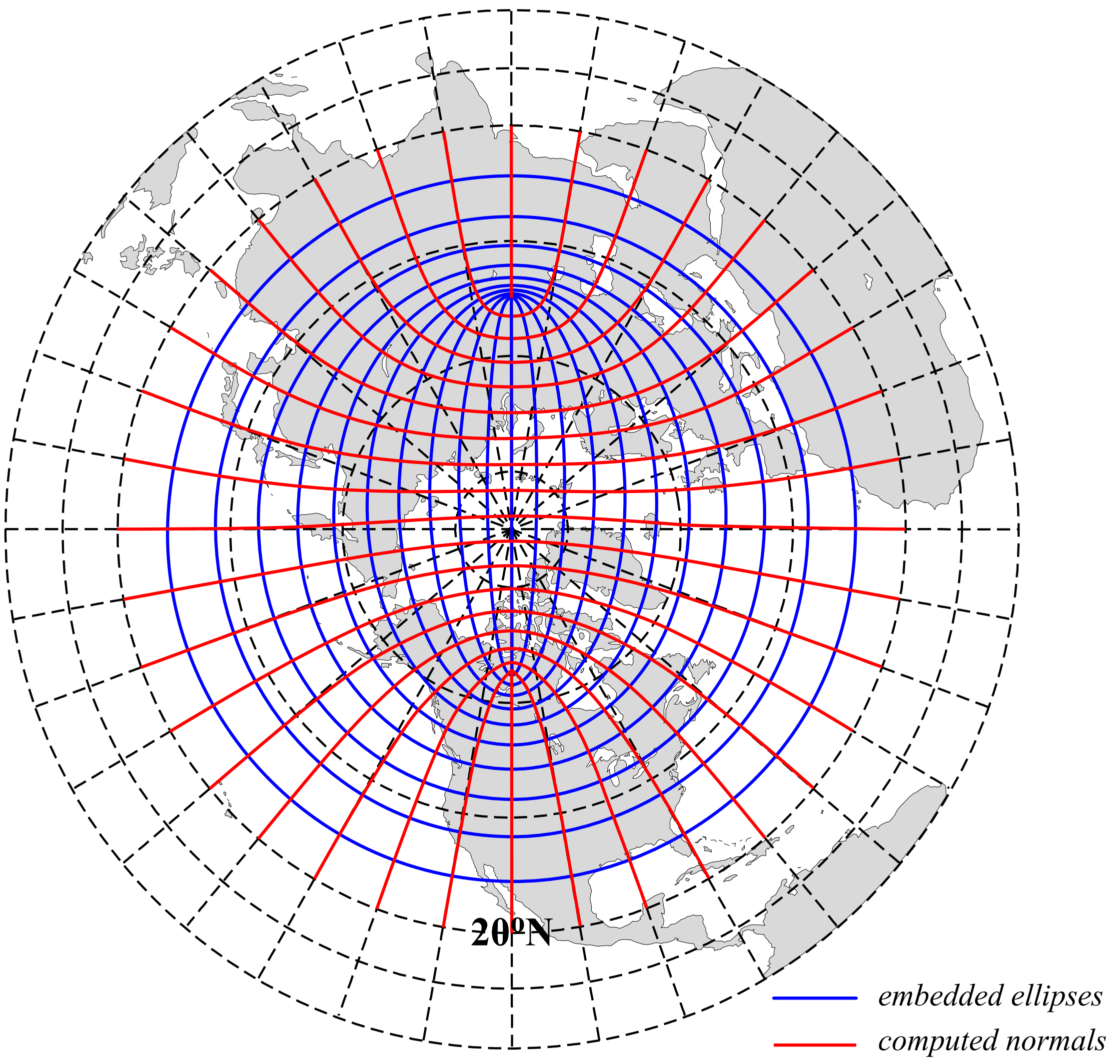
The equation of heat transfer

$$\frac{\partial T}{\partial t} = -\nabla\left(T\,\mathbf{U}\right) + D^T + F^T$$

The equation for salinity

$$\frac{\partial S}{\partial t} = -\nabla\left(S\,\mathbf{U}\right) + D^S + F^S$$

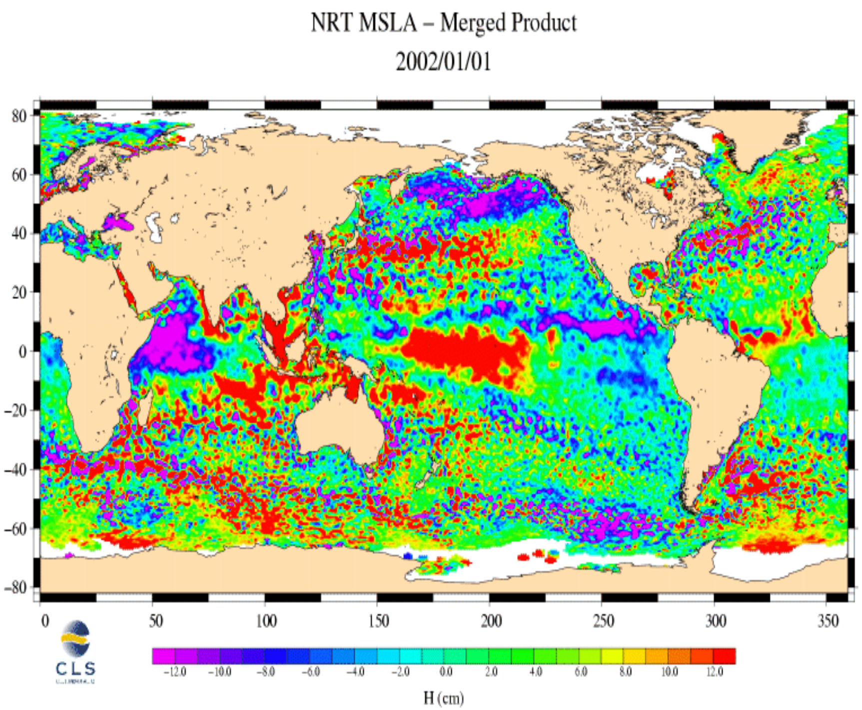
ORCA Three Pole Ocean Mesh for NEMO

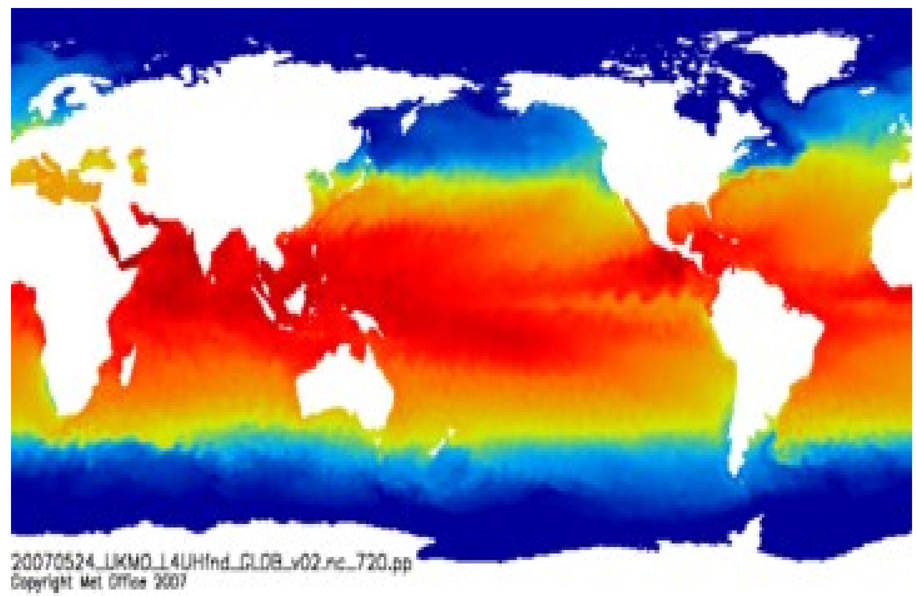


OBSERVED DATA
by satellites

Jason-2

Poseidon-3





Sea surface height (SSH)

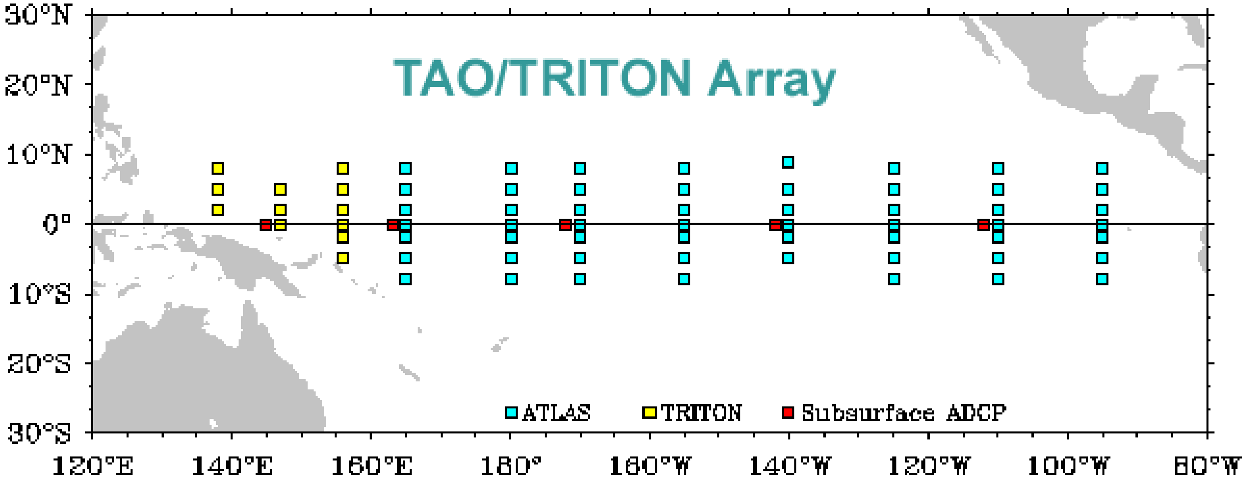
Sea surface temperature (SST)

www.aviso.oceanobs.com

OBSERVED DATA

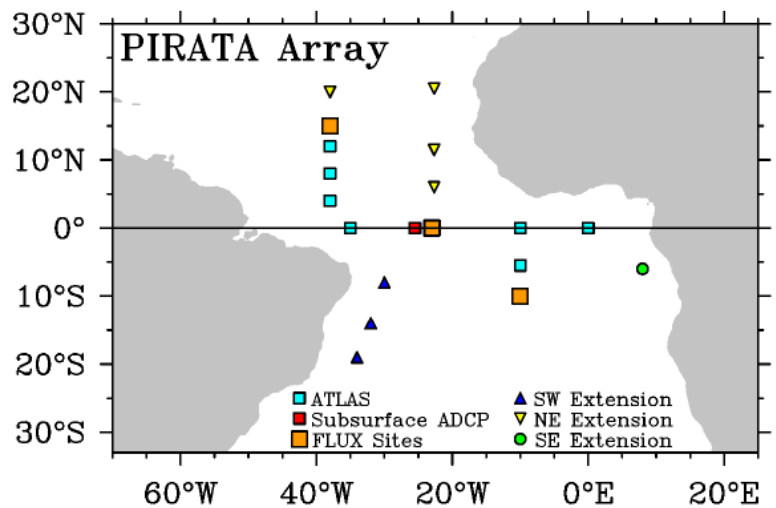
Mooring buoys (measurement of a temperature and salinity)

Tropical Atmosphere Ocean Project (TAO/TRITON)



Pacific Ocean

Pilot Research Moored Array in the Tropical Atlantic (PIRATA)

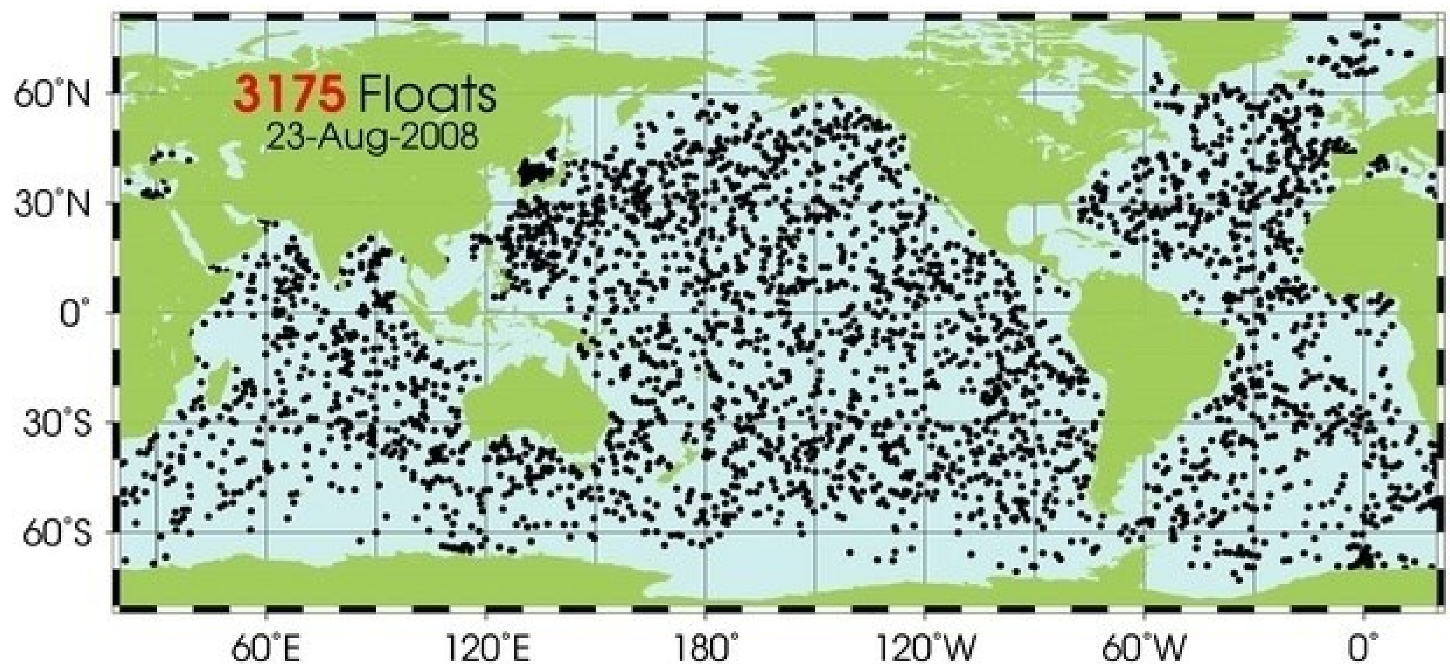


www.pmel.noaa.gov

OBSERVED DATA

Drifters Argo

Initialised in 2000



www.argo.ucsd.edu

Main assimilation model

$$X_a = X_b + K(Y - HX_b) \qquad (1)$$

X is the state vector of the ocean parameters, which includes the temperature (θ), the salinity (S), the sea surface height (SSH), the sea surface height anomaly (SSHA) (is the difference between the SSH and the long-term average at the same points), at all

X_b is the background state vector

X_a is the analysis state vector – the ocean parameters after assimilation

Y is the vector of the observable parameters $\dim Y < \dim X$

H is the projection operator matrix

K is Kalman gain matrix (Kalman filter)

A new method for constructing the gain matrix K, which provides better data assimilation was developed.

Data assimilation method

GKF (Generalized Kalman Filter)

$$\frac{\partial X}{\partial t} = \Lambda(X,t) \qquad (1)$$

$$X(0) = X_0$$

$$0 \le t \le T \qquad 0 = t_0 < t_1 < .... < t_N = T, \quad t_{n+1} = t_n + \Delta t$$

$$X_{a,n} = X_{b,n} + K_n(Y_n - HX_{b,n}) \qquad (2)$$

$$K_{n+1} = (\sigma_{n+1}^2)^{-1}(\Lambda_{n+1} - C_{n+1})(H\Lambda_{n+1})^{\mathrm{T}} Q_{n+1}^{-1} \qquad (3)$$

$$\sigma_{n+1}^2 = (H\Lambda_{n+1})^{\mathrm{T}} Q_{n+1}^{-1}(H\Lambda_{n+1}) \qquad (4)$$

The change of the state vector in conjunction with observational data can be written as a stochastic process of the following type [*K. Belyaev et al.* // MCMDS, 2018]

$$dX(t) = (I - KH)\Lambda dt + (KQK')dW, \quad t_n \le t \le t_{n+1}$$

where the first term of the correction is the observational trend extrapolated over entire model space and the second term is given by the stochastic process taking into account
the variance between the results of computations and observations.

$$Q = E(Y - HX)(Y - HX)' + R$$

is the covariance matrix of the modeling error plus covariance matrix of the observations error (R),
E is the ensemble average or mathematical expectation,
dW is the standard notation of a white noise random variable,
the apostrophe ′ denotes the transpose of a vector or a matrix.

Assimilation problem may be formulated as follows:
find out the gain matrix K which will give minimum of the variance state vector X by minimizing a norm of the diffusion matrix KQK'

$$J(K) = \| KQK' \| = \mathrm{tr}(KQK') \to \min$$

under the condition that the main part of the correction is given

$$(I - KH)\Lambda = C$$

In order to solve this optimization problem, the theory of conditional extremes will be applied.
Lagrange functional L is considered:

$$\mathrm{L}(K, \lambda) = \| KQK' \| + \lambda[(I - KH)\Lambda - C]$$

λ is the unknown vector, so-called Lagrange multiplier vector

$$L(K, \varphi) \to \min$$

Solution of optimization problem

Condition of minimum of the functional $L(K,\lambda)$

$$\frac{\partial \mathrm{L}(K,\lambda)}{\partial K} = 0$$

leads to the equations:

$$KQ - \frac{1}{2}\lambda(H\Lambda)' = 0,$$

$$(I - KH)\Lambda = C$$

The solution of this system can be obtained explicitly

$$K = \frac{(\Lambda - C)(H\Lambda)' Q^{-1}}{(H\Lambda)' Q^{-1} H\Lambda} \qquad (4)$$

Model with data assimilation (DA) method

is defined by the formulas (3),(4):

$$X_{a,n+1} = F(X_{a,n}) + K_{n+1}(Y_{n+1} - HF(X_{a,n}))$$

$$K = \frac{(\Lambda - C)(H\Lambda)'Q^{-1}}{(H\Lambda)'Q^{-1}H\Lambda}$$

where H, Q can be defined for each specific task,

$$\Lambda = \frac{\partial F}{\partial t} \quad \to \quad \Lambda_{n+1} = \frac{F(X_{a,n}) - X_{a,n}}{\Delta t}, \quad C_{n+1} = \frac{E(\hat{Y}_{n+1} - \hat{Y}_n)}{\Delta t},$$

in this way $K_{n+1} = K_{n+1}(Y_{n+1}, X_{b,n+1}, X_{a,n})$.

Thus the DA method takes into account the model dynamics.

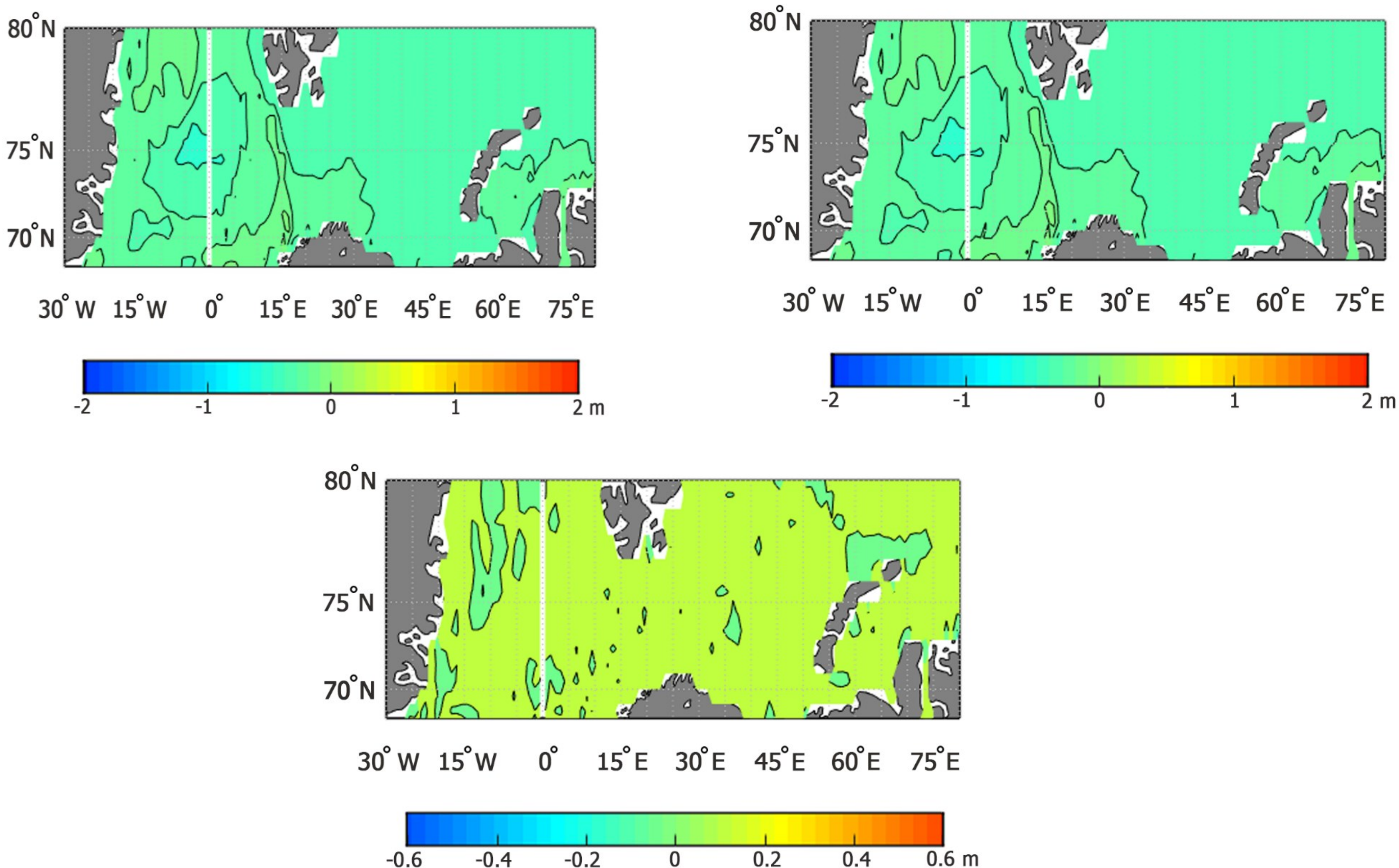
In Kalman gain matrix $K = BH(HBH' + R)^{-1}$

where $B_{n+1} = B_{n+1}(Y_{n+1}, X_{b,n+1}) \to K_{n+1} = K_{n+1}(Y_{n+1}, X_{b,n+1})$

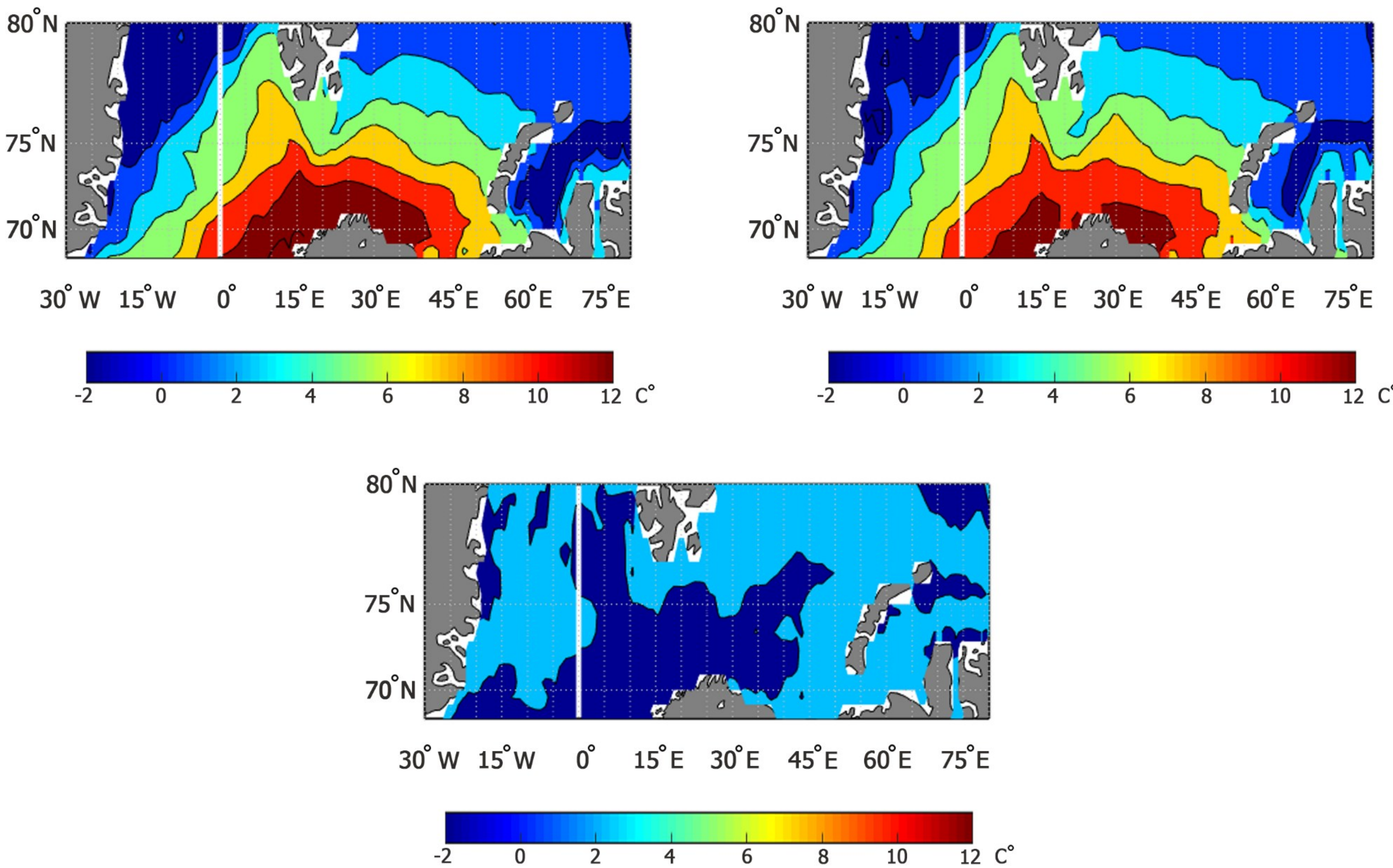
RESULTS OF THE NUMERICAL SIMULATION

using observational data from archive AVISO

Ocean level fields: (a) before DA; (b) after DA; and (c) difference between ocean levels before and after DA.



Sea Surface Temperature in Arctic, before, after correction and their difference.



Conclusion

The performed numerical experiments allow making a conclusion that the developed data assimilation method reduces the model error and leads to the results corresponding to the observed data both in quantity and in quality. Application of this method is feasible, the method produces reasonable model fields and the results can be tested numerically and compared with the natural data.